Example 10. Suppose the RREF of the augmented matrix of a system of linear equations is

$$\begin{array}{ccc} x & y & z \end{array}$$
$$\left[\begin{array}{cccc} 1 & 4 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

What is the corresponding system? What are the leading variables? What are the free variables? What are the solutions of this system?

System: $x + 4y - z = 0$ Leading variables: x
$\qquad\qquad\quad 0 = 1$ Free variables: y, z

Solutions: NONE!   System is inconsistent, since it requires $0 = 1$

- Possible outcomes for a system of linear equations:

1. unique solution   2. infinitely many solutions   3. no solutions

Example 11. Consider the following system of equations:

$$x + y - 2z = 1$$
$$-x + 10z = -1$$
$$2x + 3y + 4z = 2$$

a. Form the augmented matrix for this system.
b. Solve this system by putting its augmented matrix into RREF.

$$\left[\begin{array}{cccc} 1 & 1 & -2 & 1 \\ -1 & 0 & 10 & -1 \\ 2 & 3 & 4 & 2 \end{array}\right] \xrightarrow{R_2 + R_1} \left[\begin{array}{cccc} 1 & 1 & -2 & 1 \\ 0 & 1 & 8 & 0 \\ 2 & 3 & 4 & 2 \end{array}\right] \xrightarrow{R_3 - 2R_1} \left[\begin{array}{cccc} 1 & 1 & -2 & 1 \\ 0 & 1 & 8 & 0 \\ 0 & 1 & 8 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 - R_2} \left[\begin{array}{cccc} 1 & 1 & -2 & 1 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_1 - R_2} \begin{array}{ccc} x & y & z \end{array} \left[\begin{array}{cccc} 1 & 0 & -10 & 1 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

REF   RREF

Leading variables: x, y
Free variables: z

Solutions: $x = 1 + 10z$
$\qquad\qquad y = -8z$

In vector form: $\left[\begin{array}{c} x \\ y \\ z \end{array}\right] = \left[\begin{array}{c} 1 \\ 0 \\ 0 \end{array}\right] + z \left[\begin{array}{c} 10 \\ -8 \\ 1 \end{array}\right]$